One-loop renormalization of the Yang-Mills theory with BRST-invariant mass term.

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Abstract

Divergent part of the one-loop effective action for the Yang-Mills theory in a special gauge containing forth degrees of ghost fields and allowing addition of BRST-invariant mass term is calculated by the generalized t'Hooft-Veltman technique. The result is BRST-invariant and defines running mass, coupling constant and parameter of the gauge.

1 Introduction.

It is well known, that adding of the mass term to the action of the Yang-Mills theory breaks the gauge invariance. That is why the massive gauge fields are usually introduced by the spontaneous symmetry breaking mechanism [1]. Nevertheless, there is an alternative approach. It is based on the fact, that the quantization of Yang-Mills theory requires to fix the gauge invariance and add to the action ghost Lagrangian [2]. The total Lagrangian is then invariant under BRST transformations, which remain from the original gauge transformations. In principle, it is possible to construct mass term invariant only under these residual transformations. For a general gauge this problem is not solved. However, there is a special gauge, containing forth degree of ghost fields [3], allowing existence of the BRST-invariant mass term.

It is interesting to investigate quantum properties of such theory and, in particular, to check BRST invariance of the effective action. In principle, the one-loop correction was found in [4] by the diagram technique. However, the calculations are rather

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involved and it is desirable to check them by another method of calculation. One of the suitable calculational tools is t'Hooft-Veltman technique [5], which was originally proposed for a limited class of theories and afterwards generalized for the general case in [6]. In the frames of t'Hooft-Veltman method the divergent part of the one-loop effective action can be calculated by some purely algebraic operations. In this paper such calculation is made for the Yang-Mills theory with the BRST-invariant mass term.

The paper is organized as follows: Some information about Yang-Mills theory with BRST-invariant mass term is reminded in Section 2. The generalized t'Hooft-Veltman technique and the calculation process is briefly described in Section 3. Renormalization of the model is discussed in Section 4. In this section we also check, that the divergent part of the one-loop effective action is invariant under BRST transformations. The obtained results are briefly discussed in Conclusion and some technical details of calculations are presented in the Appendix.

2 Yang-Mills theory with BRST-invariant mass term.

If the gauge invariance in the Yang-Mills theory is fixed by adding of the following terms [3]

$$S_{gf} + S_{gh} = \frac{1}{e^2}\mathrm{tr}\int d^4x\Big(\alpha B^2 - 2\partial_\mu B\, A^\mu - i\alpha B\{c\,\bar c\} - 2i\partial_\mu\bar c\mathcal{D}^\mu c + \alpha c^2\bar c^2\Big), \qquad (1)$$

then it is possible to add a mass term, invariant under BRST transformations up to a total derivative. The action of the obtained theory is written as

$$\begin{aligned} S = \frac{1}{e^2}\mathrm{tr}\int d^4x\Big(\frac{1}{2}F_{\mu\nu}^2 + \alpha B^2 - 2\partial_\mu B\, A^\mu - i\alpha B\{c\,\bar c\} - \\ -2i\partial_\mu\bar c\mathcal{D}^\mu c + \alpha c^2\bar c^2 - m^2(A_\mu^2 - 2i\alpha c\bar c)\Big). \end{aligned} \qquad (2)$$

and is invariant under the following BRST transformations:

$$\begin{aligned} &\delta_b A_\mu(x) = \varepsilon_b\mathcal{D}_\mu c(x);\\ &\delta_b c(x) = -\varepsilon_b c(x)^2;\\ &\delta_b\bar c(x) = i\varepsilon_b B(x);\\ &\delta_b B(x) = 0. \end{aligned} \qquad (3)$$

There is also an invariance under anti-BRST transformations

$$
\begin{aligned}
&\delta_a A_\mu(x) = \varepsilon_a \mathcal{D}_\mu \bar{c}(x);\\
&\delta_a c(x) = -\varepsilon_a (iB(x) + \{c(x), \bar{c}(x)\});\\
&\delta_a \bar{c}(x) = -\varepsilon_a \bar{c}(x)^2;\\
&\delta_a B(x) = 0.
\end{aligned}
\tag{4}
$$

3 Calculation of the divergent part of the one-loop effective action.

For calculation of divergent part of the effective action it is possible to use different methods, for example, diagram technique or t'Hooft-Veltman method [5, 6]. In this paper we will use the second method. Let us remind its main ideas:

It is well known [1], that for a theory described by a classical action $S(\varphi_i)$ the one-loop effective action can be written as

$$
\Gamma^{(1)} = \frac{i}{2}\mathrm{Str}\ \ln\ D_i{}^j \tag{5}
$$

where

$$
D_i{}^j = \frac{\delta^2 S}{\delta\varphi^i \delta\varphi_j} \tag{6}
$$

is an operator of second variation of the classical action. In the most general case this operator can be written as

$$
\begin{aligned}
D_i{}^j =\ & K^{\mu_1\mu_2\ldots\mu_L}{}_i{}^j\ \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_L} +\ S^{\mu_1\mu_2\ldots\mu_{L-1}}{}_i{}^j\ \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_{L-1}}\\
& +\ W^{\mu_1\mu_2\ldots\mu_{L-2}}{}_i{}^j\ \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_{L-2}} +\ N^{\mu_1\mu_2\ldots\mu_{L-3}}{}_i{}^j\ \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_{L-3}}\\
& +\ M^{\mu_1\mu_2\ldots\mu_{L-4}}{}_i{}^j\ \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_{L-4}} + \ldots,
\end{aligned}
\tag{7}
$$

where all tensors are considered to be symmetrical with respect to permutations of Lorentz indexes and ∇_μ denotes a covariant derivative

$$
\begin{aligned}
&\nabla_\alpha T^\beta{}_i{}^j = \partial_\alpha T^\beta{}_i{}^j + \Gamma^\beta_{\alpha\gamma} T^\gamma{}_i{}^j + \omega_{\alpha i}{}^k T^\beta{}_k{}^j - T^\beta{}_i{}^k \omega_{\alpha k}{}^j;\\
&\nabla_\mu \Phi_i = \partial_\mu \Phi_i + \omega_{\mu i}{}^j \Phi_j,
\end{aligned}
\tag{8}
$$

$\Gamma^\alpha_{\mu\nu}$ is a Cristofel symbol

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}(\partial_\mu g_{\nu\beta} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu}) \tag{9}$$

and $\omega_{\mu i}{}^{j}$ is a connection in the principle bundle.

The divergent part of one-loop effective action (5) for operator (7) was found explicitly in [6]. In the particular case, when operator (7) has second order in derivatives ($L = 2$), and the space-time is flat, the result is written as

$$\begin{aligned}\Gamma^{(1)}_{\infty} = &-\frac{1}{16\pi^2}\ln\frac{M}{\mu}\,\mathrm{Str}\int d^4x \Big\langle \frac{1}{2}\hat{W}^2 - \hat{W}\,\hat{S}^2 - 2\partial_\mu\hat{S}\,\hat{W}\,\hat{K}^\mu + \\ &+\frac{1}{4}\hat{S}^4 + \frac{1}{3}\Big(\partial_\mu\hat{S}^\mu\hat{S}^2 - 2\partial_\mu\hat{S}\,\hat{K}^\mu\hat{S}^2 - \partial_\mu\hat{S}\,\hat{S}\,\hat{S}^\mu + 2\partial_\mu\hat{S}\,\hat{S}^2\hat{K}^\mu\Big) + \\ &+\partial_\mu\hat{S}\,\partial_\nu\hat{S}^\nu\hat{K}^\mu - \frac{1}{2}\partial_\mu\hat{S}\,\partial_\nu\hat{S}\left(-\hat{K}^{\mu\nu} + 2\hat{K}^\mu\hat{K}^\nu + 2\hat{K}^\nu\hat{K}^\mu\right)\Big\rangle,\end{aligned} \tag{10}$$

where the notations can be explained by the following equations:

$$\begin{aligned} &\hat{S}_i{}^j \equiv (Kn)^{-1}{}_i{}^k (Sn)_k{}^j; &&\hat{K}^\mu{}_i{}^j \equiv (Kn)^{-1}{}_i{}^k (Kn)^\mu{}_k{}^j;\\ &(Sn)_i{}^j \equiv S^{\mu_1\mu_2\ldots\mu_{L-1}}{}_i{}^j n_{\mu_1}n_{\mu_2}\ldots n_{\mu_{L-1}}; &&(Kn)^\mu{}_i{}^j \equiv K^{\mu\mu_2\ldots\mu_L}{}_i{}^j n_{\mu_2}\ldots n_{\mu_L};\\ &&(Kn)_i{}^k (Kn)^{-1}{}_k{}^j = \delta^j_i.\end{aligned} \tag{11}$$

Here n_μ is a unit vector and the angle brackets denotes the following operation:

$$\begin{aligned}\langle n_{\mu_1}n_{\mu_2}\ldots n_{\mu_{2m}}\rangle \equiv &\frac{1}{2^m(m+1)!}\\ &\times\Big(g_{\mu_1\mu_2}\,g_{\mu_3\mu_4}\ldots g_{\mu_{2m-1}\mu_{2m}} + \text{permutations }(\mu_1\ldots\mu_{2m})\Big).\end{aligned} \tag{12}$$

Second variation of action (2), which defines matrixes K, S and W, is presented in the Appendix. The matrixes constructed ¿from it were substituted to equation (10), which gives the divergent part of the one-loop effective action. After this operation we obtained the following result:

$$\begin{aligned}\Gamma^{(1)}_{\infty} = &\frac{c_2}{8\pi^2}\,\ln\frac{M}{\mu}\,\mathrm{tr}\int d^4x\Bigg(\frac{1}{2}\Big(-\frac{\alpha}{2}-\frac{13}{6}\Big)\Big(\partial_\mu A^R_\nu - \partial_\nu A^R_\mu\Big)^2 + \Big(-\frac{3\alpha}{4}-\frac{17}{12}\Big)\times\\ &\times\Big(\partial_\mu A^R_\nu - \partial_\nu A^R_\mu\Big)\Big(A^R_\mu A^R_\nu - A^R_\nu A^R_\mu\Big) + \frac{1}{2}\Big(-\alpha-\frac{2}{3}\Big)\Big(A^R_\mu A^R_\nu - A^R_\nu A^R_\mu\Big)^2 -\\ &-\frac{\alpha^2}{4}(B^R)^2 + \frac{\alpha}{2}\partial_\mu B^R\,A^R_\mu + i\frac{\alpha^2}{2}B^R\{c^R\,\bar{c}^R\} + \frac{\alpha-3}{4}m^2(A^R_\mu)^2 - i\frac{\alpha^2}{2}m^2c^R\,\bar{c}^R -\\ &-\frac{3}{4}\alpha^2(c^R)^2(\bar{c}^R)^2 + i\frac{\alpha+3}{2}\partial_\mu\bar{c}^R\,\partial_\mu c^R + i\alpha\,\partial_\mu\bar{c}^R\,[A^R_\mu, c^R]\Bigg)\end{aligned} \tag{13}$$

4 One-loop renormalization ans BRST-invariance of the effective action.

After adding counterterms, corresponding to equation (13) to the classical action (2), the renormalized action can be written as

$$S_{ren} = S + \Delta S = \frac{1}{e_0^2}\mathrm{tr}\int d^4x\Big(\frac{1}{2}F_{\mu\nu}^2 + \alpha_0 B^2 - 2\partial_\mu B\, A^\mu -$$
$$-i\alpha_0 B\{c\,\bar{c}\} - 2i\partial_\mu\bar{c}\,\mathcal{D}^\mu c + \alpha_0 c^2\bar{c}^2 - m_0^2(A_\mu^2 - 2i\alpha_0 c\bar{c})\Big) \tag{14}$$

where

$$\begin{aligned}
\frac{1}{e^2} &= \frac{1}{e_0^2} - \frac{1}{8\pi^2}c_2\,\frac{11}{3}\ln\frac{M}{\mu} + O(e_0^2);\\
m &= m_0 + \frac{e_0^2}{8\pi^2}c_2\ln\frac{M}{\mu}\,m_0\Big(\frac{\alpha_0}{8} + \frac{35}{24}\Big) + O(e_0^4);\\
\alpha &= \alpha_0 - \frac{e_0^2}{8\pi^2}c_2\ln\frac{M}{\mu}\Big(\frac{\alpha_0^2}{4} + \frac{13\alpha_0}{6}\Big) + O(e_0^4),
\end{aligned} \tag{15}$$

and the renormalized fields are defined by the equations

$$\begin{aligned}
A_\mu^R &= \left[1 - \frac{e_0^2}{8\pi^2}c_2\ln\frac{M}{\mu}\frac{(\alpha_0 - 3)}{4} + O(e_0^4)\right]A_\mu;\\
\bar{c}^R\cdot c^R &= \left[1 - \frac{e_0^2}{8\pi^2}c_2\ln\frac{M}{\mu}\left(\frac{\alpha_0}{4} - \frac{35}{12}\right) + O(e_0^4)\right]\ \bar{c}\cdot c;\\
B^R &= \left[1 + \frac{e_0^2}{8\pi^2}c_2\ \frac{35}{12}\ln\frac{M}{\mu} + O(e_0^4)\right]B.
\end{aligned} \tag{16}$$

Because renormalized action (14) has exactly the same structure as original action (2), it is invariant under BRST transformations (3) and anti-BRST transformations (4).

5 Conclusion.

In this paper we found the divergent part of the one-loop effective action for the Yang-Mills theory with BRST-invariant mass term. To perform this calculation we used a method, which was originally proposed by t'Hooft and Veltman [5] and afterwards generalized in [6] for more complicated cases. The considered theory turns out to be renormalizable and BRST-invariant at the quantum level. The results obtained for running mass and parameter of the gauge coincide with the corresponding results, found in [4] by the diagram technique up to notations. Therefore, the calculation confirms correction of results of [4] and also correctness of rather complicated algorithms, constructed in [6].

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A Appendix.

Second variation of action (2) up to a multiplicative constant can be written as

$$D_i{}^j = \begin{pmatrix} d_{AA} & d_{Ac} & d_{A\bar{c}} \\ d_{cA} & d_{cc} & d_{c\bar{c}} \\ d_{\bar{c}A} & d_{\bar{c}c} & d_{\bar{c}\bar{c}} \end{pmatrix} \tag{17}$$

where

$$\begin{aligned}
d_{AA} &= \Big(\mathcal{D}_\alpha^2 + m^2\Big)\eta^{\mu\nu} - \mathcal{D}^\nu\mathcal{D}^\mu - \frac{1}{\alpha}\partial^\mu\partial^\nu + \mathbf{F}^{\mu\nu};\\
d_{Ac} &= \frac{i}{2}\bar{\mathbf{c}}\,\partial^\mu - \frac{i}{2}(\partial^\mu\bar{\mathbf{c}});\\
d_{A\bar{c}} &= -\frac{i}{2}\mathbf{c}\,\partial^\mu + \frac{i}{2}(\partial^\mu\mathbf{c});\\
d_{cA} &= -\frac{i}{2}\bar{\mathbf{c}}\,\partial^\nu - i(\partial^\nu\bar{\mathbf{c}});\\
d_{cc} &= \frac{\alpha}{4}\bar{\mathbf{c}}^2;\\
d_{c\bar{c}} &= i\mathcal{D}_\alpha\partial^\alpha - \frac{i\alpha}{2}\mathbf{B} - i\alpha m^2 - \frac{\alpha}{4}\bar{\mathbf{c}}\mathbf{c} - \frac{\alpha}{2}\mathbf{c}\bar{\mathbf{c}};\\
d_{\bar{c}A} &= \frac{i}{2}\mathbf{c}\,\partial^\nu + i(\partial^\nu\mathbf{c});\\
d_{\bar{c}c} &= -i\partial_\alpha\mathcal{D}^\alpha - \frac{i\alpha}{2}\mathbf{B} + i\alpha m^2 - \frac{\alpha}{4}\mathbf{c}\bar{\mathbf{c}} - \frac{\alpha}{2}\bar{\mathbf{c}}\mathbf{c};\\
d_{\bar{c}\bar{c}} &= \frac{\alpha}{4}\mathbf{c}^2.
\end{aligned} \tag{18}$$

Here we used the following notations:

$$\mathbf{B} \equiv -ieB^aT^a, \qquad (T^a)_{bc} = -if_{abc}, \quad \text{etc.} \tag{19}$$

Coefficients at the second derivatives in equation (18) form the matrix K, coefficients at the first derivatives form the matrix S and terms without derivatives form the matrix W.

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